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Coset models and D-branes in group manifolds

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Abstract

We conjecture the existence of a duality between heterotic closed strings on homogeneous spaces and symmetry-preserving D-branes on group manifolds, based on the observation about the coincidence of the low-energy field description for the two theories. For the closed string side we also give an explicit proof of a no-renormalization theorem as a consequence of a hidden symmetry and infer that the same property should hold true for the higher order terms of the DBI action.

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One of the main technical advantages provided by the study of models on group manifolds is that the geometrical analysis can be recast in Lie algebraic terms. At the same time the underlying conformal symmetry makes it possible to explicitly study the integrability properties that, in general, allow for extremely nice behaviours under renormalization. Wess-Zumino-Witten models can be used as starting points for many interesting models: the main challenge in this case consists in partially removing the symmetry while retaining as many algebraic and integrability properties as possible.

In this note we aim at pointing out an analogy (or, as we will say, a duality) between two – in principle disconnected – constructions based on WZW models: closed string (heterotic) backgrounds obtained via asymmetric deformations and symmetry-preserving D-branes on group manifolds. As we will show, in fact, the low-energy field contents for both theories are the same, although they minimize different effective actions (SUGRA for the former and DBI for the latter). For one of the sides of the duality (the closed string one) we will also show a no-renormalization theorem stating that the effect of higher-order terms can be resummed to a shift in the radii of the manifold. A similar behaviour can also be conjectured from the D-brane side, and this would be consistent with some remark in literature about the coincidence between the DBI and CFT results concerning mass spectra, ... up to the said shift [1, 2].

Let us start with the open-string side of this duality, by reminding some known facts about the geometric description of D-branes in WZW models on compact groups, pointing out in particular the low-energy field configuration. Natural boundary conditions on WZW models are those in which the gluing between left- and right-moving currents can be expressed in terms of automorphisms ω of the current algebra. The corresponding world-volumes are then given by (twisted) conjugacy classes on the group [3]:

$$\mathcal{C}^{\omega}(g) = \left\{ \, hg\omega(h^{-1}) \,\middle|\, h \in G \, \right\} . \tag{1}$$

As it was pointed out in [1], one can use Weyl's theory of conjugacy classes so to give a geometric description of $C^{\omega}(g)$. For a given automorphism ω we can always find an ω-invariant maximal torus $T \subset G$ (such as $\omega(T) = T$). Let $T^{\omega} \subset T$ be the set of elements $t \in T$ invariant under ω ($T^{\omega} = \{ \, t \in T \,|\, \omega(t) = t \, \}$) and $T_0^{\omega} \subset T^{\omega}$ the connected component to the unity. When ω is inner $T = T^{\omega} = T_0^{\omega}$ while in general (*i.e.* if we allow ω to be outer) $\dim(T_0^{\omega}) \leq \operatorname{rank} G$.

Let ω be inner. Define a map:

$$\begin{aligned} q : G/T \times T \to G \\ ([g], t) \mapsto q([g], t) = gtg^{-1}. \end{aligned} \tag{2}$$

One can show that this map is surjective, so that each element in G is conjugated to some element in T. This implies in particular that the conjugacy classes are characterized by elements in T, or, in other words, fixing $t \in T$ (so to take care of the action of the Weyl group), we find that the (regular) conjugacy classes $\mathcal{C}^\omega(g)$ are isomorphic to the homogeneous space G/T. A similar result holds for twisted classes, but in this case

$$\mathcal{C}^\omega(g) \simeq G/T_0^\omega. \tag{3}$$

The description of the D-brane is completed by the $U(1)$ gauge field that lives on it. The possible $U(1)$ fluxes are elements in $H^2(G/T_0^\omega, \mathbb{R})$ and one can show that

$$H^2(G/T_0^\omega) \simeq \mathbb{Z}^{\dim T_0^\omega}. \tag{4}$$

Summarizing we find that the gauge content of the low energy theory is given by:

- the metric on G/T_0^ω (in particular G/T for untwisted branes),
- the pull-back of the Kalb–Ramond field on G/T_0^ω,
- $\dim T_0^\omega$ independent $U(1)$ fluxes ($\operatorname{rank} G$ for untwisted branes).

These fields extremize the DBI action

$$S = \int \mathrm{d}x \sqrt{\det\left(\mathbf{g} + \mathbf{B} + 2\pi \mathbf{F}\right)}. \tag{5}$$

and according to some coincidence with known exact CFT results there are reasons to believe that the fields only receive a normalization shift when computed at all loops.

Let us now move to the other – closed string – side of the advertised duality. A good candidate for a deformation of a WZW model that reduces the symmetry, at the same time preserving the integrability and renormalization properties, is obtained via the introduction

of a truly marginal operator written as the product of a holomorphic and an antiholomorphic current

$$\mathcal{O} = \sum_{ij} c_{ij} J^i \bar{J}^j. \tag{6}$$

As it was shown in [4], a necessary and sufficient condition for this marginal operator to be integrable is that the left and right currents both belong to abelian groups. If we consider the heterotic super-WZW model, a possible choice consists in taking the left currents in the Cartan torus and the right currents from the heterotic gauge sector [5, 6]:

$$\mathcal{O} = \sum_{a=1}^{N} \mathrm{H}_a J^a \bar{I}^a. \tag{7}$$

where $J^a \in H \subset T$, T being the maximal torus in G.

Using a construction bearing many resemblances to a Kaluza–Klein reduction it is straightforward to show that the background fields corresponding to this kind of deformation consist in a metric, a Kalb–Ramond field and a $U(1)^N$ gauge field. Their explicit expressions are simply given in terms of Maurer–Cartan one-forms on G as follows:

$$g = \frac{k}{2}\delta_{\mathrm{MN}} \mathcal{J}^{\mathrm{M}} \otimes \mathcal{J}^{\mathrm{N}} - k\delta_{ab}\mathrm{H}_a^2 \tilde{\mathcal{J}}^a \otimes \tilde{\mathcal{J}}^b, \tag{8a}$$

$$H_{[3]} = \mathrm{d}B - \frac{1}{k_g} A^a \wedge \mathrm{d}A^a = \frac{k}{2} f_{\mathrm{MNP}} \mathcal{J}^{\mathrm{M}} \wedge \mathcal{J}^{\mathrm{N}} \wedge \mathcal{J}^{\mathrm{P}} - k\mathrm{H}_a^2 f_{a\mathrm{MN}} \mathcal{J}^a \wedge \mathcal{J}^{\mathrm{M}} \wedge \mathcal{J}^{\mathrm{N}}, \tag{8b}$$

$$A^a = \mathrm{H}_a \sqrt{\frac{2k}{k_g}} \tilde{\mathcal{J}}^a \text{ (no summation over } a \text{ implied)}, \tag{8c}$$

where $\tilde{\mathcal{J}}^a_\mu$ are the currents that have been selected for the deformation operator. In this way we get an N-dimensional space of exact models. Here we will concentrate on a special point in this space, namely the one that corresponds to $\{ \mathrm{H}_a = 1/\sqrt{2}, \forall a = 1, 2, \dots, N \}$. This point is remarkable for it corresponds to a decompactification limit where N dimensions decouple and we're left with the homogeneous G/H space times N non-compact dimensions*. More

* One can see the initial group manifold G as a principal fibration of H over a G/H basis: the deformation changes the radii of the fiber and eventually trivializes in correspondence of this special point.

precisely, when H coincides with the maximal torus T, the background fields read:

$$G = \frac{k}{2} \sum_{\mu} \mathcal{J}^{\mu} \otimes \mathcal{J}^{\mu}, \tag{9a}$$

$$H_{[3]} = \mathrm{d}B = \frac{1}{2} f_{\mu\nu\rho} \mathcal{J}^{\mu} \wedge \mathcal{J}^{\nu} \wedge \mathcal{J}^{\rho}, \tag{9b}$$

$$F^{a} = -\sqrt{\frac{k}{2k_g}} \mathrm{H}_a f^{a}_{\ \mu\nu} \mathcal{J}^{\mu} \wedge \mathcal{J}^{\nu} \tag{9c}$$

(no summation over a). Geometrically:

- g is the metric on G/T obtained as the restriction of the Cartan–Killing metric on G
- $H_{[3]}$ is the pullback of the usual Kalb–Ramond field present in the WZW model on the group G
- F^a are rank(G) independent $U(1)$ gauge fluxes that satisfy some quantization conditions and hence naturally live in $H^2(G/T, \mathbb{Z})$

Having chosen a truly marginal operator for the deformation we know that this model is conformal. This implies in particular that the background fields solve the usual β equations that stem from the variation of the effective SUGRA action:

$$S = \int \mathrm{d}x \sqrt{g} \left(R - \frac{1}{12} H_{\mu\rho\sigma} H^{\mu\rho\sigma} - \frac{k_g}{8} F^{a}_{\mu\nu} F^{a\mu\nu} + \frac{\delta c}{3} \right) \tag{10}$$

In example if we consider $G = SU(2)$, then $T = U(1)$ and the decompactification limit $H \to 1/\sqrt{2}$ we get the exact $S^2 = SU(2)/U(1)$ background supported by a $U(1)$ magnetic monopole field (see *e.g.* [7, 8]).

Our conjecture stems precisely from this: the gauge field above exactly match the ones we found before for symmetry-preserving D-branes. Moreover both sides of the duality are derived from WZW models that enjoy a no-renormalization property which would make this correspondence true at all orders. In this spirit we now pass to prove that a similar theorem holds for closed heterotic strings on coset models infering that the duality, when proven, would give a direct way to deduce the same feature for the D-brane action.

In studying symmetrically deformed WZW models, *i.e.* those where the deformation operator is written as the product of two currents belonging to the same sector $\mathcal{O} = \lambda J \bar{J}$, one finds that the Lagrangian formulation only corresponds to a small-deformation approximation. For this reason different techniques have been developed so to read the background

fields at every order in λ [9, 10, 11, 12, 13] but, still, the results are in general only valid at first order in α' and have to be modified so to take into account the effect of instanton corrections. In this section we want to show that this is not the case for asymmetrically deformed models, for which the background fields in Eqs. (8) are exact at all orders in H_a and for which the effect of renormalization only amounts to the usual (for WZW models) shift in the level of the algebra $k \to k + c_G$ where c_G is the dual Coxeter number.

Consider in example the most simple $SU(2)$ case. In terms of Euler angles the deformed Lagrangian is written as:

$$S = S_{SU(2)}(\alpha,\beta,\gamma) + \delta S = \frac{k}{4\pi}\int \mathrm{d}^2 z \; \partial\alpha\,\bar\partial\alpha + \partial\beta\,\bar\partial\beta + \partial\gamma\,\bar\partial\gamma + 2\cos\beta\,\partial\alpha\,\bar\partial\gamma + \\ + \frac{\sqrt{k k_g}\mathrm{H}}{2\pi}\int \mathrm{d}^2 z \; \left(\partial\gamma + \cos\beta\,\partial\alpha\right)\bar{I}. \quad (11)$$

If we bosonize the right-moving current as $\bar{I} = \bar\partial\phi$ and add a standard $U(1)$ term to the action, we get:

$$S = S_{SU(2)}(\alpha,\beta,\gamma) + \delta S(\alpha,\beta,\gamma,\phi) + \frac{k_g}{4\pi}\int \mathrm{d}^2 z \; \partial\phi\,\bar\partial\phi = \\ = S_{SU(2)}\left(\alpha,\beta,\gamma + 2\sqrt{\frac{k_g}{k}}\mathrm{H}\phi\right) + \frac{k_g\left(1 - 2\mathrm{H}^2\right)}{4\pi}\int \mathrm{d}^2 z\,\partial\phi\,\bar\partial\phi \quad (12)$$

and in particular at the decoupling limit $\mathrm{H} \to 1/\sqrt{2}$, corresponding to the S^2 geometry, the action is just given by $S = S_{SU(2)}\left(\alpha,\beta,\gamma + 2\sqrt{\frac{k_g}{k}}\mathrm{H}\phi\right)$. This implies that our (deformed) model inherits all the integrability and renormalization properties of the standard $SU(2)$ WZW model. In other words the three-dimensional model with metric and Kalb–Ramond field with $SU(2)\times U(1)$ symmetry and a $U(1)$ gauge field is uplifted to an exact model on the $SU(2)$ group manifold (at least locally): the integrability properties are then a consequence of this hidden $SU(2)\times SU(2)$ symmetry that is manifest in higher dimensions.

The generalization of this particular construction to higher groups is easily obtained if one remarks that the Euler parametrization for the $g \in SU(2)$ group representative is written as:

$$g = \mathrm{e}^{\imath\gamma t_3}\mathrm{e}^{\imath\beta t_1}\mathrm{e}^{\imath\alpha t_2}, \quad (13)$$

where $t_i = \sigma_i/2$ are the generators of $\mathfrak{su}(2)$ (σ_i being the usual Pauli matrices). As stated above, the limit deformation corresponds to the gauging of the left action of an abelian

subgroup $T \subset SU(2)$. In particular here we chose $T = \{ h \mid h = e^{\imath \phi t_3} \}$, hence it is natural to find (up to the normalization) that:

$$h(\phi)\, g(\alpha, \beta, \gamma) = g(\alpha, \beta, \gamma + \phi)\,. \tag{14}$$

The only thing that one needs to do in order to generalize this result to a general group G consists in finding a parametrization of $g \in G$ such as the chosen abelian subgroup appears as a left factor. In example if in $SU(3)$ we want to gauge the $U(1)^2$ abelian subgroup generated by $\langle \lambda_3, \lambda_8 \rangle$ (Gell-Mann matrices), we can choose the following parametrization for $g \in SU(3)$ [14]:

$$g = e^{\imath \lambda_8 \phi} e^{\imath \lambda_3 c} e^{\imath \lambda_2 b} e^{\imath \lambda_3 a} e^{\imath \lambda_5 \vartheta} e^{\imath \lambda_3 \gamma} e^{\imath \lambda_2 \beta} e^{\imath \lambda_3 \alpha}. \tag{15}$$

The deep reason that lies behind this property (differentiating symmetric and asymmetric deformations) is the fact that not only the currents used for the deformation are preserved (as it happens in both cases), but here their very expression is just modified by a constant factor. In fact, if we write the deformed metric as in Eq. (8a) and call $\tilde{K}^\mu$ the Killing vector corresponding to the chosen isometry (that doesn't change along the deformation), we see that the corresponding $\tilde{\mathcal{J}}_\mu^{(\mathrm{H})}$ current is given by:

$$\tilde{\mathcal{J}}_\nu^{(\mathrm{H})} = \tilde{K}^\mu g_{\mu\nu}^{(\mathrm{H})} = \left(1 - 2\mathrm{H}^2\right) \tilde{\mathcal{J}}_\nu^{(0)} \tag{16}$$

The most important consequence (from our point of view) of this integrability property is that the SUGRA action in Eq. (10) is *exact* and the only effect of renormalization is the $k \to k + c_G$ shift.

It is very tempting to exend this no-renormalization theorem to the D-brane side. Of course this would require an actual proof of the duality we conjecture. Nevertheless we think that this kind of approach might prove (at least for these highly symmetric systems) more fruitful than adding higher loop corrections to the DBI action, which on the other hand remains an interesting directions of study by itself.

Is this duality just a coincidence, due to the underlying Lie algebraic structures that both sides share, or is it a sign of the presence of some deeper connection? Different aspects of the profound meaning of the DBI effective action are still poorly understood and it is possible that this approach – pointing to one more link to conformal field theory – might help shedding some new light.

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